

84.937 Connecting Mathematical Concepts and Teaching

Syllabus

Instructor: Dr. Arnold Good
Office Hours: ½ hour before and after class
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Supportive Texts: Elementary Number Theory with Applications; T. Koshy; Harcourt
About Teaching Mathematics; M. Burns; Math Solutions
A History of Mathematics; C. Boyer; Wiley
The Lady Tasting Tea ; D. Salsburg; Owl Books

Course Description

A course designed for teachers to investigate the major mathematical concepts and content found in the Massachusetts Mathematics Curriculum Frameworks in order to improve their understanding and recognition of connections within the mathematics curriculum. By analyzing classroom cases, participants learn to identify mathematical concepts with which students struggle. Teachers improve their ability to communicate mathematical ideas to students.

Grading: 800 points	
300pts	Class Participation: 100 pts per Section
200 pts	Total value of short quizzes administered at the beginning of class sessions
100 pts	Number Sense subject matter exam
100 pts	Algebra subject matter exam
100 pts	Probability and Statistics subject matter exam

Explanation of Why this is a Graduate Course

There are two reasons why this is a graduate mathematics course.

(1) Many topics are covered from an abstract, general, applications point of view. For example the Greatest Common Divisor is not only found, it is treated abstractly, applied to Diophantine equations, and used in establishing certain number-theoretic formulas. Arithmetic operations are generalized and applied to situations involving infinity. Participants are introduced to the concept of the existence of a solution prior to attempts to find a solution. Algebraic expressions for geometric figures are not treated individually but are compared to each other with an attempt to develop classifications. Consideration is given to Dandelin Spheres, and the Theorems of Pascal and Brianchon. The geometric principle of duality is explained. Statistics is presented using an axiomatic framework atypical of an undergraduate approach to the subject. The connection between statistics and calculus is also explored.

(2) Participants are expected to make connections between course topics and what they teach in their own classrooms. Consequently, professional status as a school teacher is a prerequisite to the course. As a course for professionals, which addresses issues confronting professionals, this course transcends the ordinary undergraduate preparatory course for the prospective school teacher.

Course Philosophy

Mathematics text books currently on the market and recommended for graduate-level courses for elementary school teachers turn out to be those also used in undergraduate teacher-education mathematics courses. These texts address topics that teachers teach to their own elementary-level students – topics which the teacher has been exposed to in their own elementary education, in their own high school education, and in their own undergraduate college-level mathematics preparation. Covering these topics again at the graduate level has questionable value. On the other hand, teaching teachers new ways to *teach* mathematics may be worthwhile but this would be part of a mathematics methods course which this course is not.

The purpose of this course is to expose teachers to topics beyond what is taught to elementary school children. In other subjects teachers know what advanced material looks like. Teachers may teach basic reading skills, but teachers read at far more advanced levels themselves. But in teaching mathematics, teachers know only what they teach. This course attempts to change that. To do so it has to divert from the traditional way that mathematics is taught. Normally, mathematics is taught in a problem solving manner. The instructor does a problem at the board and the student is expected to duplicate the technique on homework and exams. Few other subjects are taught this way. Imagine a literature course covering the novel *Moby Dick* in which the instructor tells the student at the completion of the material that they must now write their own novel about a great white whale. Because it is recognized that elementary school teachers may not be able to do advanced mathematics, no one tries to teach teachers advanced mathematics. What if English teachers said that since elementary teachers cannot be expected to write Broadway plays they should not be taught Shakespeare!

This course covers advanced topics in a lecture format similar to many humanities courses. Topics have been simplified so that teachers can appreciate them even if they themselves cannot do the advanced mathematics presented. Homework and exam problems do not oblige the teacher to do advanced mathematics. This approach to teaching mathematics could give teachers a richer idea of what mathematics is all about and motivate them to do a better job at teaching lower-level math to their own elementary level students. Motivated teachers can then advance their mathematics teaching skills through thought methods-directed professional education courses for PDPs

Course Outline

Number Sense

Elementary Number Theory: Parts of Chapters 1, 2, 3 and 8.

About Teaching Mathematics: Parts IV (last part of text)

A History of Mathematics: Parts of Chapter 1 though 6.

Short quizzes will begin Sessions 2, 3, 4, and 5

- Ses 1 (Unit 3): An historical introduction to the natural numbers beginning with the contribution of the Near East.
Prime Numbers and Godel's Enumeration
Infinite Series and Decimal Representations
- Ses 2 (Unit 7): Greatest Common Divisor and the Euclidean Algorithm
Euler's Phi Function; the Tau Function
Consideration of the Locked-Doors Problem
- Ses 3 (Unit 5): Modular Arithmetic, Cyclic Groups, and Fermat's Little Theorem
Consideration of the King Arthur Problem
- Ses 4 (Unit 9): Review of Area Formulas and the Pythagorean Theorem
Consideration of the Tangram and failure of the six-piece square
- Ses 5 (Unit 11): Mathematical Topics using a Geoboard
Introduction to Topology; Euler's Formula; the 4-Color Problem
Linear Diophantine Equation and its Geometric Interpretation
Areas of Polygons using Interior and Perimeter Vertices
- Ses 6 (Unit 13): One hour exam
Length and enclosed area of Koch Curve/Snow Flak
- Mentoring: Mentoring sessions, analyzing classroom cases, will be conducted immediately following short quizzes and prior to the introduction of new mathematics content. Mathematics for students with special needs will be considered during these mentoring sessions
- To the extent possible, mentoring topic will reflect content topic covered in previous session and be suitable to the grade-level of the participant's classroom.
- Discussion on how topics might be adjusted for various learning needs or to reflect cultural diversity issues will be undertaken.

Algebra and Problem Solving

Elementary Number Theory: Parts of Chapter 3
A History of Mathematics: Parts of Chapters 13, 14, and 15.

Short quizzes will begin Sessions 2, 3, 4, and 5.

- Ses 1 (Unit 16): An historical introduction to algebra beginning with the contribution of the Arabic World
The equation of a line, slope of a line, system of equations
Introduction to Differential Calculus
- Ses 2 (Unit 18): Linear Programming with applications
- Ses 3 (Unit 20): Introduction to the general quadratic equation
Algebraic consequences of planar translations and rotations
- Ses 4 (Unit 22): Introduction to Conics Sections: Ellipse, Parabola, Hyperbola
Algebraic interpretations of conics using Dandelin spheres
- Sec 5 (Unit 24): Introduction to the Theorems of Pascal and Brianchon
Discussion of the geometric Principle of Duality
[Session topic may be replaced by a session on the use of mathematics manipulatives where a guest presenter will be present](#)
- Ses 6 (Unit 26): One hour exam
Demonstration that the parabola is an infinitely expanded ellipse
- Mentoring: Mentoring sessions, analyzing classroom cases, will be conducted immediately following short quizzes and prior to the introduction of new mathematics content. Mathematics for students with special needs will be considered during these mentoring sessions
- To the extent possible, mentoring topic will reflect content topic covered in previous session and be suitable to the grade-level of the participant's classroom.
- Discussion on how topics might be adjusted for various learning needs or to reflect cultural diversity issues will be undertaken.

Probability and Statistics

The Lady Tasting Tea: Pages 1 through 93

Short quizzes will begin Sessions 2, 3, 4, 5 and 6.

- Ses 1 (Unit 29): An historical introduction to statistics beginning with the contribution of the Europeans
Discussion of Peano's Axioms and Axioms for Statistics
Organizing data
- Ses 2 (Unit 31): Metric Space approach to Standard Deviation (Mean is found first)
Comparing two sets of data using Regression Line and Correlation
- Ses 3 (Unit 33): Factorials and Advanced Counting Principles
Elementary Probability Theory. Expected Value.
- Ses 4 (Unit 35): Examples of Probability Distributions using Curves
Determining Areas and Introduction to Integral Calculus
Standard Normal Random Variable values using Tables
- Ses 5 (Unit 37): The Central Limit Theory
Application to estimating parameters (population means)
- Ses 6 (Unit 29) One hour exam
Use of statistics in decision making problems
- Mentoring: Mentoring sessions, analyzing classroom cases, will be conducted immediately following short quizzes and prior to the introduction of new mathematics content. Mathematics for students with special needs will be considered during these mentoring sessions
- To the extent possible, mentoring topic will reflect content topic covered in previous session and be suitable to the grade-level of the participant's classroom.
- Discussion on how topics might be adjusted for various learning needs or to reflect cultural diversity issues will be undertaken.

Course Resources

As this is a *graduate* level course for elementary school teachers, finding a suitable textbook has not proved possible. Traditional textbooks designed for the elementary school teacher – such as Mathematics for Elementary School Teachers by Billstein, Liberskind, and Lott; or Mathematical Ideas by Miller, Heeren, and Hornsby – are undergraduate level in nature and focus primarily on the mathematics that elementary school teachers teach their own students. As participants of this course are already practicing teachers, it is assumed that they have already completed a course for which these traditional texts have been designed. Consequently, resources for this course are found on the World Wide Web or will be distributed as handouts by the course instructor. These resources addresses topics beyond those covered in traditional undergraduate courses and which convey to the participating teacher a better sense of the nature and excitement of mathematical invention and discovery.

Electronic resources can readily be found using conventional search engines such as Google. Some available resources are:

Prime numbers: www-gap.dcs.st-and.ac.uk/history/HistTopics/Prime_numbers.html

GCD: mathworks.wolfram.com/GreatestCommonDivisor

Modular Arithmetic: cut-the-knot.org/Curriculum/Algebra/Modulo.shtml

Cyclic Groups: mathworld.wolfram.com/CyclicGroupC4.html

Pythagoreans Theorem: cut-the-knot.org/Pythagoras/index.shtml

Tangrams: standards.nctm.org/documents/examples/chap4/4.4.html

Geoboards: standards.nctm.org/documents/examples/chap4/4.2.html

Four Color Problem: math.gatech.edu/~thomas/FC/fourcolor.html

Diophantine Equations: mathworks.wolfram.com/DiophantineEquations.html

Koch Curve: zeuscat.com/andrea/chaos/vonkoch.html

Algebra/History: www.ucs.louisiana.edu/~sxw8045

Equation of a Line: id.mind.net/~zona/mmts/functionInstitute/linearFunction/lsif.html

Quadratic Equation: mathworld.wolfram.com/QuadraticsEquations.html

Conic Sections: www.math2.org/math/algebra/conics.html

Conic Sections: mathworld.wolfram.com/ConicSections.html

Brianchon Theorem: www.geom.uiuc.edu/apps/conics/conic2.html

Duality Principle: www.arth.org.uk/NCT/basics.htm

Peano's Axioms: mathworld.wolfram.com/PeanosAxioms.htm

Probability: www.mathgoodies.com/lessons/toc_vol6.shtm

Statistics: math.uc.edu/~brycw/classes/147/blue/topics.htm

Class Participation

Participating teachers are expected to attend all class sessions and to take an active role in each class session. Homework problems assigned at the end of each session are to be completed prior to the next class session when they will be discussed. In addition, participants will be asked to develop and use lesson plans covering topics similar to what is being covered in class. The effectiveness of these lesson plans will be discussed during the mentoring sessions preceding the mathematics content portion of the course.

Mentoring Component

Activities undertaken during mentoring sessions will be directed by the particular topics participants decide to cover in their own classrooms. Mentoring will address lesson plans teachers develop that are similar to course topics while appropriate for their grade level. To the extent possible, items addressed during mentoring sessions will be commented upon during content sessions so that participants grasp the connection between what they are teaching their students and what they themselves are being taught. No course credit will be given for this part of the course. However, the involvement of participants in these sessions will be part of the overall assessment made by the Program Advisor who represents the sponsoring organization.

Assessment

The assessment of participating teachers will be done using class participation, quizzes, and exams on course content. The numerical value of each component is stated in the earlier table. In addition, an ongoing overall assessment will be undertaken when the course instructor and the individual representing the sponsoring organization (advisor) meet. Class participation, journal entries, quizzes/exams scores, and portfolio development would be taken into consideration during these meetings. Teachers not performing well will be counseled-out of the program.

Additional Topics for Middle School Mathematics Teachers

Quaternions Algebra and Calculus; David Eberly

Quaternions; Mathworld.Wolfram.com

Chaos by James Cleick

Handouts

These following topics have been chosen because they involve material either taught at the upper-middle school level or early in high school. They involve an excellent review of topics which should be familiar to the middle school teacher but provide either unexpected historical insight or modern applications. The plan is to provide modestly advanced topics that will prove to be very interesting and informative to the middle school mathematics teacher.

- Section 1: Review of complex numbers and matrix algebra
 Review of vectors including dot and cross products
 Introduction to quaternions and their little known historical role in
 the formulation of topics covered at the middle and high school
 level
- Section 2: Presentation of chaos theory and fractal geometry beginning with
 an introduction to iteration sequences based on linear and
 quadratics equations. Discussion using simple theorems from
 calculus and fixed point concepts are included. Use of the
 geometry of straight lines as well as parabolas will be considered.
- Exam: Online exam at end of course.